

Malati

Mathematics learning and teaching initiative

ALGEBRA

MODULE 8

QUADRATIC EQUATIONS

GRADE 9

TEACHER DOCUMENT

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Introduction and overview of Module 8

In this module:

- We will revisit the notion of an equation in the context of quadratic equations.
- Pupils will be encouraged to use their own informal methods before being introduced to formal solution procedures.
- We will revisit the concept (meaning) of the solution of an equation. The number of solutions of an equation (no solution; 1 solution, 2 solutions or many solutions) will be dripped.
- Out of this we will extract the notion of a quadratic equation, so as to distinguish it from linear and other equations.
- Finally, we will reflect on the solution procedures.

Assumptions:

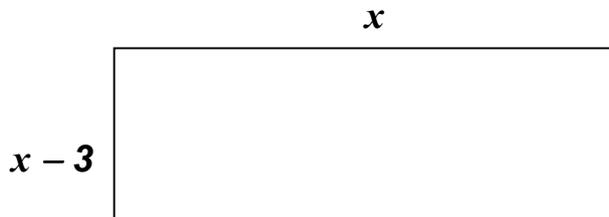
1. that pupils know how to factorise trinomials and complete the square
2. that pupils are familiar with the meaning of "square" and the concept of "perfect square".

ACTIVITY 1

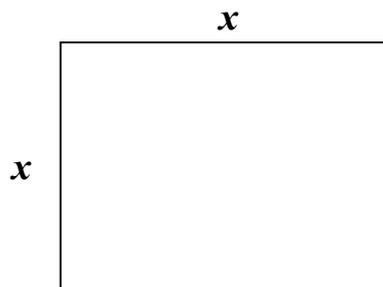
A gardener wants his garden to have an interesting geometrical appearance. He decides on the following rules for building the flowerbeds

- ❖ They must all be rectangular.
- ❖ The perimeter and the area must be the same.

1. How many different flowerbeds can the gardener make if one of the sides is 3 units less than the other side as shown in the diagram below:



2. How many different flowerbeds can the gardener make if both sides are the same length, as shown in the diagram below:



Teacher Notes: Activity 1

The aim of this activity is to model a situation that leads to the quadratic equations:

1. $x(x - 3) = 4x - 6$

2. $x^2 = 4x$

The pupils need to use their own strategies to solve these equations. The pupils may, for example, establish a set of equivalent quadratic equations through the balancing method that they are familiar with in the context of linear equations:

1. $x^2 - 3x = 4x + 6 \Rightarrow x^2 - 7x = -6$

2. $x^2 - 4x = 0$

However, pupils will probably have no other method available but to solve these equations using numerical methods (setting up a table or proceeding with guess and improve). The pupils might set up tables from the original equations:

x	$x(x - 3)$	$4x - 6$
3	0	6
4	4	10

or

$x^2 - 7x$	-6
-12	-6
-12	-6

The pupils need to be encouraged to move through the numbers to find the solutions and to make sense of the solution in the context of the problem.

It also needs to be made explicit here that we are now dealing with an equation that involves a term with an unknown of **the second degree**. This is one feature that distinguishes it from linear equations previously dealt with.

Note: In using the balancing method, pupils might very well divide both sides of equation $x^2 = 4x$ by x , obtaining a partial solution $x = 4$. This must be discussed.

ACTIVITY 2

The gardener embarks on another plan for creating the geometrical appearance of his garden. He decides on the following rules for building the pair of flowerbeds:

- ◆ They must all be rectangular.
- ◆ The one flowerbed must be larger than the other flowerbed.

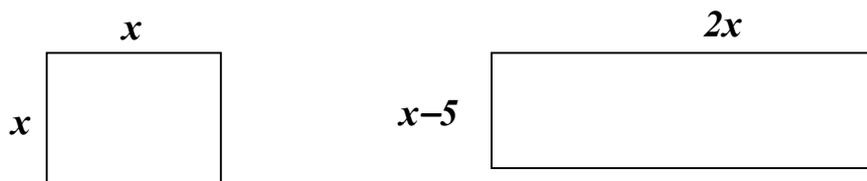
How many different pairs of flowerbeds can the gardener create and what is the size of these flowerbeds if:

1. One of the flowerbeds is a square and the other flowerbed has one of its sides equal to the side of the square and a side that is 10 units as shown in the diagram below:



The square flowerbed is 24 square units smaller than the other flowerbed.

2. One of the flowerbeds is a square and the other flowerbed has one of its sides twice the length of the square flowerbed and the other side 5 units smaller than the length of the square as shown in the diagram below:

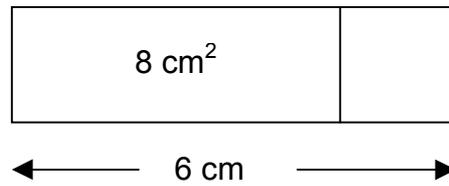


The area of the square flowerbed is 24 units larger than the area of the other flowerbed.

Teacher Notes: Activities 2 & 3

The aim of these activities is to model a situation that leads to a quadratic equation in which both solutions are valid in the context of the problem. Allow the pupils to use their own strategies.

In Activity 2 the drawings assist the pupils in modeling the situation. In Activity 3 the drawing (below) is not included, to allow pupils to model the situation using their own strategies.



In the whole-class discussion the teacher needs to ensure that the quadratic model is also represented.

ACTIVITY 3

By putting a rectangle of area 8 cm^2 right next to a square, we obtain a rectangle of length 6 cm. How long is the square?

ACTIVITY 4

Larry takes a number and subtracts 5 from it. He takes the same number and subtracts 17 from it. Larry takes these two new numbers and multiplies them. The result he gets is 0.

1. Write an equation that will represent the story. (Use the letter x to represent the number that Larry takes.)
2. What number did Larry take?

ACTIVITY 5

Larry takes a number and squares it. Larry takes the same number again but now multiplies it by 5 and subtracts the answer from the square of the number. He gets an answer of 24.

What number did Larry take?

ACTIVITY 6

Larry takes a number and squares it. He subtracts 16 from this new number and gets 0.

What number did Larry take?

Teacher Notes: Activities 4, 5 & 6

In the class discussion the quadratic model must be discussed also.

Ensure that the pupils realise that they have to have **both** solutions and that one of the solutions is not the correct solution.

ACTIVITY 7

These are equations that we created from the situations in Activities 1–6:

GROUP A
$x(x-3) = 4x - 6$
$x^2 - 3x = 4x - 6$
$x^2 = 4x$
$10x - x^2 = 24$
$2x^2 - 10x + 24 = x^2$
$(x - 5)(x - 12) = 0$
$x^2 - 17x + 60 = 0$
$x^2 - 5x = 24$
$x(6 - x) = 8$
$x^2 - 16 = 0$

These are some equations we worked with in previous modules:

GROUP B
$12 - 4x = 3x - 9$
$2x - 3(3 + x) = 5x + 9$
$3x + 5 = 41$
$7x - 3 + 2x + 5 = 0$

Add another two equations to each group.

What is the differences between the two groups of equations?

How do you distinguish between the two groups of equations?

Teacher Notes: Activity 7

Whole-class discussion:

Let pupils reflect on the difference between the two groups of equations. The criteria suggested by the pupils should be discussed. The criteria must be valid for the group of equations created. For example, if a pupil says that a quadratic equation must have x^2 and x as terms, it can be pointed out that some of the equations do not have x , for example, $(x^2 - 16 = 0)$. The teacher can also point out that for now we are dealing with equations with only one unknown. The teacher can refer to the table and point out that the unknown is x .

The discussion can be extended to include the categorisation of equations on the basis of the number of solutions. It needs to be pointed out to the pupils that traditionally, in mathematics, the categorisation of equations is on the basis of the maximum number of solutions that is determined by **the degree** of the unknown.

The pupils can also connect the equations to its graphical representation using the graphics calculator.

Now a definition of the linear and the quadratic equation with one unknown can be introduced. For example, we can describe linear equations with **one unknown** as equations that contain only first powers of the unknown. We can describe quadratic equations with **one unknown** as equations that contain second powers of the unknown.

ACTIVITY 8

Sort the following equations with one unknown into the following categories:

linear; quadratic and neither.

$$2x(x - 3) = 8 \quad 5x = 3x + 8 \quad x + 1 = x^2 + 13$$

$$3x(x - 2) = 4(x + 1) \quad x^2(3 - x) + 7 = x + x^2 + 2 \quad 2x^2 - (2x^2 - 5x + 7) = 3$$

$$3x(4x - x^2 + 2) = x^2(4x + 3)$$

LINEAR	QUADRATIC	NEITHER

ACTIVITY 9

A gardener builds flowerbeds according to the following rules:

- ✓ They must all be rectangular
- ✓ The sides are all whole numbers
- ✓ The area is equal to the perimeter

How many flowerbeds can he build?

ACTIVITY 10

There are 63 chairs in a room, neatly arranged in 6 long rows and 3 short rows. How many chairs are in a long row and how many chairs are in a short row?

Teacher Notes: Activities 9 &10

Allow the pupils to use their own strategies. The whole-class discussion should lead the situation being modeled as equations involving two unknowns:

Activity 9: $xy = 2x + 2y$

Activity 10: $6x + 3y = 63$

We can now revisit the definition of linear and quadratic equations involving two unknowns.

ACTIVITY 11

Solve the following quadratic equations:

1. $x^2 + 7 = 23$

2. $x^2 + 3x = 0$

3. $x^2 + 23 = 7$

4. $2x^2 - 3x + 1 = 0$

5. $(2x - 1)^2 = 4$

6. $x^2 + 5x + 6 = 0$

Teacher Note: Activity 11

First allow pupils to use their own strategies for solving the equations.

In the class discussion the pupils need to be explicitly made aware that they have specific tools such as factorising, using squares and square roots to solve some of the equations. The limitations of these tools become evident in question 4.

The discussion at this point should not lead to the generalisation of solving quadratic equations, but to focus instead on the solution **procedures** of the given generic examples. The generalisation of the solution procedures of the different subsets will be dealt with in the next activity. The different subsets are:

1. $ax^2 + b = 0$
2. $ax^2 + bx = 0$
3. $(a_1x + b_1)(a_2x + b_2) = 0$
4. $(ax + b)^2 = c$
5. $ax^2 + bx + c = 0$

ACTIVITY 12

All the equations below can be solved using techniques in the previous activity. Analyse the *structure* of the equations and justify on the basis of the structure how it can be solved.

1. $x^2 + 7 = 3x^2$

2. $x^2 - 7 = 3x^2 - 169$

3. $5x = x^2$

4. $24x^2 - 38x + 15 = 0$

5. $(2x - 1)^2 = 0$

6. $(2x - 1)^2 = 4$

7. $(2x - 1)^2 + 4 = 0$

8. $(2x - 3)^2 = x^2$

9. $x^2 - 14x + 40 = 0$

10. $x^2 - 120 = -14x$

Teacher Notes: Activity 12

The aim of this activity is to allow the pupils to focus on the **structure** of the given quadratic equations so as to recognise the different subgroups.

In the class discussion the structures of the different subgroups can be generalised.

In the following activities we will deal with the solution procedures of the different subgroups.

ACTIVITY 13

Solve the following equations:

1. $2x^2 - 7 = 25$

2. $x^2 + 3 = 12$

3. $3(x^2 + 1) = 30$

4. $2x^2 + 1 = 76$

5. $2x^2 + 25 = 7$

6. $2(x^2 + 3) - 4 = 20$

What is the general structure of these equations?

ACTIVITY 14

Solve the following equations:

1. $(x - 1)^2 = 9$

2. $3(x + 1)^2 = 30$

3. $2(x + 3)^2 - 4 = 20$

4. $-3(x + 1)^2 + 4 = 22$

5. $\frac{(x + 4)^2 - 4}{15} = 4$

6. $\left(\frac{x + 4}{15}\right)^2 - 4 = 0$

What is the general structure of these equations?

ACTIVITY 15

Solve the following equations:

1. $x^2 + 4x = 0$

2. $3x^2 = 2x$

3. $5x = -2x^2 - 3x$

4. $4x^2 = 3x^2 + x$

5. $3x = 2x(x + 1)$

What is the general structure of these equations?

ACTIVITY 16

Solve the following equations:

1. $x^2 + 6x + 9 = 0$

2. $x^2 - 4x + 4 = 0$

3. $x^2 + 10x = -25$

4. $4x^2 = 12x - 9$

What is the general structure of these equations?

Teacher Notes: Activities 13, 14 ,15 & 16

In the class discussion point out that within these structures it is possible to have different numbers of solutions, for example:

$2x^2 + 25 = 7$ has no solution.

$x^2 + 6x + 9 = 0$ has one solution

$x^2 + 4x = 0$ has two solutions

ACTIVITY 17

Solve the following equations:

1. $x^2 + 6x + 5 = 0$

2. $3x^2 + 2x - 2 = 0$

3. $6x^2 - x - 1 = 0$

4. $2x^2 - 12 = 8x + 2$

5. $x^2 + 3x = 8 - 2x$

What is the general structure of these equations?

Were you able to solve all the equations?

Teacher Notes: Activity 17

In the class discussion the general structure $ax^2 + bx + c = 0$ should be dealt with. For now it needs to be made explicit that equations which do not belong to the subgroup $(ax_1 + b_1)(ax_2 + b_2) = 0$ will not be dealt with now.

OVERVIEW (Whole-class setting)

We will now explicitly deal with the standard form $ax^2 + bx + c = 0$.

The discussion should examine the nature of the different coefficients a , b and c .

Let's start with b and c equal to 0. This leads to the equation $ax^2 = 0$.

Now reflect on how these equations were solved.

Consider the equations with only $b = 0$. This leads to the equation $ax^2 + c = 0$

Now reflect on how these equations were solved.

Consider the equations with only $c = 0$. This leads the equation $ax^2 + bx = 0$.

Now reflect on how these equations were solved.

Finally consider the equations where b and c were not equal to 0.

This leads to equations that:

1. can be expressed as a perfect square
2. we know how to factorise
3. we do not know how to factorise due to the inconvenient numbers.

ACTIVITY 18

Solve the following quadratic equations by factorisation:

1. $2x^2 + 5x = 3$

2. $6x^2 + 13x + 6 = 0$

3. $3x^2 = 6x + 9$

4. $3x^2 = x^2 + x + 3$

5. $2x^2 + x - 1 = 6x^2 - 4$

ACTIVITY 19

Without solving, mark which of the following equations can be written as a perfect square:

1. $9x^2 + 6x + 1 = 0$

2. $9x^2 + 18x = -1$

3. $4x^2 = 12x - 9$

4. $2x^2 - 8x = x^2 - 16$

5. $4x^2 + 8x + 4 = -4x - 5$

ACTIVITY 20

Solve the following equations:

1. $x^3 + 2x^2 = 5x$

2. $x^3 + 6x = 5x^2$

3. $2x^2 + 5 = 5 - x^3$

4. $2(x^3 - 1) + x(x^3 - 1) = 0$

5. $y + yx = 5 + 5x$

6. $x(y^2 - 1) + 5(y^2 - 1) = 0$

7. $(x^2 - 4)^2 = 9$

8. $x^4 + x^3 = 0$

9. $2x^3 - x^2 = 12x - 3x^2$

10. $x^2 + 2xy + y^2 = 0$

Teacher Notes: Activity 20

The aim of this activity is to give the pupils equations that they can solve by factorisation methods that they are familiar with. After the first stage of factorisation they can then continue with the procedures for solving quadratic equations that they know.